## Time: 3 hrs.

Max. Marks:100

## Note: Answer any FIVE full questions.

Fourth Semester B.E. Degree Examination, June/July 2017

Advanced Mathematics – II

- 1 a. Find the angle between any two diagonals of a cube. (06 Marks)
  - b. Find the angle between two lines whose direction cosines are given by  $\ell + 3m + 5n = 0$  and  $2mn 6n\ell 5\ell m = 0$ . (07 Marks)
  - c. Find the coordinates of the foot of the perpendicular from A(1, 1, 1) to the line joining the points B(1, 4, 6) and C(5, 4, 4). (07 Marks)
- 2 a. Find the equation of the plane through (2, -1, 6) and (1, -2, 4) and perpendicular to the plane x 2y 2z + 9 = 0.
  - b. Find the equation of a straight line through (7, 2, -3) and perpendicular to each of the lines.  $\frac{x-1}{3} = \frac{y-3}{4} = \frac{z-4}{5}$  and  $\frac{x+2}{4} = \frac{y-3}{5} = \frac{z-4}{6}$ . (07 Marks)
  - c. Find the angle between the planes x-y+z-6=0 and 2x+3y+z+5=0. (07 Marks)
- 3 a. If  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  are any three vectors then prove that  $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c}) \vec{b} (\vec{a} \cdot \vec{b}) \vec{c}$ (06 Marks)
  - b. If  $\vec{A} = 4i + 3j + k$ ,  $\vec{B} = 2i + j + 2k$  find a unit vector N perpendicular to the vectors  $\vec{A}$  and  $\vec{B}$  also show that  $\vec{A}$  is not perpendicular to  $\vec{B}$ .
  - c. Find the value of  $\lambda$  so that the points A(-1, 4, -3), B(3, 2, -5), C(-3, 8, -5) and D(-3,  $\lambda$ , 1) lie on the same plane. (07 Marks)
- 4 a. A particle moves along the curve  $x = 2t^2$ ,  $y = t^2 4t$ , z = 3t 5 where t is time. Find the components of its velocity and acceleration in the direction of the vector i 3j + 2k at t = 1.
  - b. Find the angle between tangents to the curve  $x = t^2 + 1$ , y = 4t + 3,  $z = 2t^2 + 6t$  at t = 1 and t = 2. (07 Marks)
  - c. Find the directional derivative of  $x^2yz + 4xz^2$  at (1, -2, -1) in the direction of 2i j 2k.

    (07 Marks)
- 5 a. Prove that  $div(curl \overrightarrow{A}) = 0$ . (06 Marks)
  - b. Find the divergence and curl of the vector.
  - $\overrightarrow{F} = (xyz + y^2z)i + (3x^2y + y^2z)j + (xz^2 y^2z)k$ c. Find the constants a, b, c so that the vector,
  - F = (x + 2y + az)i + (bx 3y z)j + (4x + cy + 2z)k is irrotational. (07 Marks)

## **MATDIP401**

c. 
$$L \left[ \frac{1 - e^{at}}{t} \right]$$
 (05 Marks)

d. 
$$L\left[\int_{0}^{t} e^{2t} \frac{\sin at}{t} dt\right]$$
 (05 Marks)

7 a. Find 
$$L^{-1} \left[ \frac{2s-1}{s^2 + 2s + 17} \right]$$
. (05 Marks)

b. Find 
$$L^{-1} \left[ \frac{s+1}{(s-1)^2(s+2)} \right]$$
. (05 Marks)

c. Find 
$$\Gamma^{-1} \left[ \cot^{-1} \left( \frac{s}{a} \right) \right]$$
. (05 Marks)

d. Using convolution theorem evaluate 
$$L^{-1}\left[\frac{s}{(s+2)(s^2+9)}\right]$$
. (05 Marks)

a. Using Laplace transforms, solve  $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = \sin t$  given y(0) = y'(0) = 0. (10 Marks)

b. Using Laplace transforms, solve  $\frac{dx}{dt} + y = \sin t$ ,  $\frac{dy}{dt} + x = \cos t$ , given x = 2, y = 0 when t = 0. (10 Marks)